ELECTROMAGNETICS WITHOUT EQUATIONS

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ELECTROMAGNETICS ISN'T POPULAR?

At the first meeting of a required undergraduate fields course which I was assigned to teach while a visiting Professor in the Electrical Engineering Department at Ohio University in Athens, I asked the 35 or so students in attendance "How many of you would be taking this course if it were not required?" When not a single hand was raised, I could tell that this was not likely to be a "fun" course to teach, nor evidently, to take.

Given the ubiquity in modern life and technology of electrostatic, magnetostatic, and electromagnetic fields, why should this be the case? The answer can probably be found not very far into most of the texts used to introduce students to a topic that arguably forms the foundation of electrical and computer engineering. Calculus, not just differential and integral, but its vector form too, soon engulfs anyone having the apparent misfortune of being led to the electromagnetics water trough. Given that most of the formative discoveries in electromagnetics were made in a rather nonmathematical context, coming from various laboratory experiments, this concentration on a mathematically intensive teaching approach to the subject seems unnecessary, especially for those who don't intend on majoring in the discipline. At the least it can be counterproductive.

The purpose of this brief discussion is to demonstrate that one of the most fundamental properties of electromagnetic (EM) fields, the radiation and propagation of power through space, can be explained without resorting to complicated mathematics. Since EM fields would not exist were it not for the phenomenon of radiation, this means that understanding EM need not be as mathematically intimidating as it's made to appear. This demonstration depends on two basic observations.

The Propagation Speed of EM Fields is Finite--EM fields propagate at the finite speed of approximately 3 x 10^8 m/s in free space, a quantity conventionally denoted as "c."

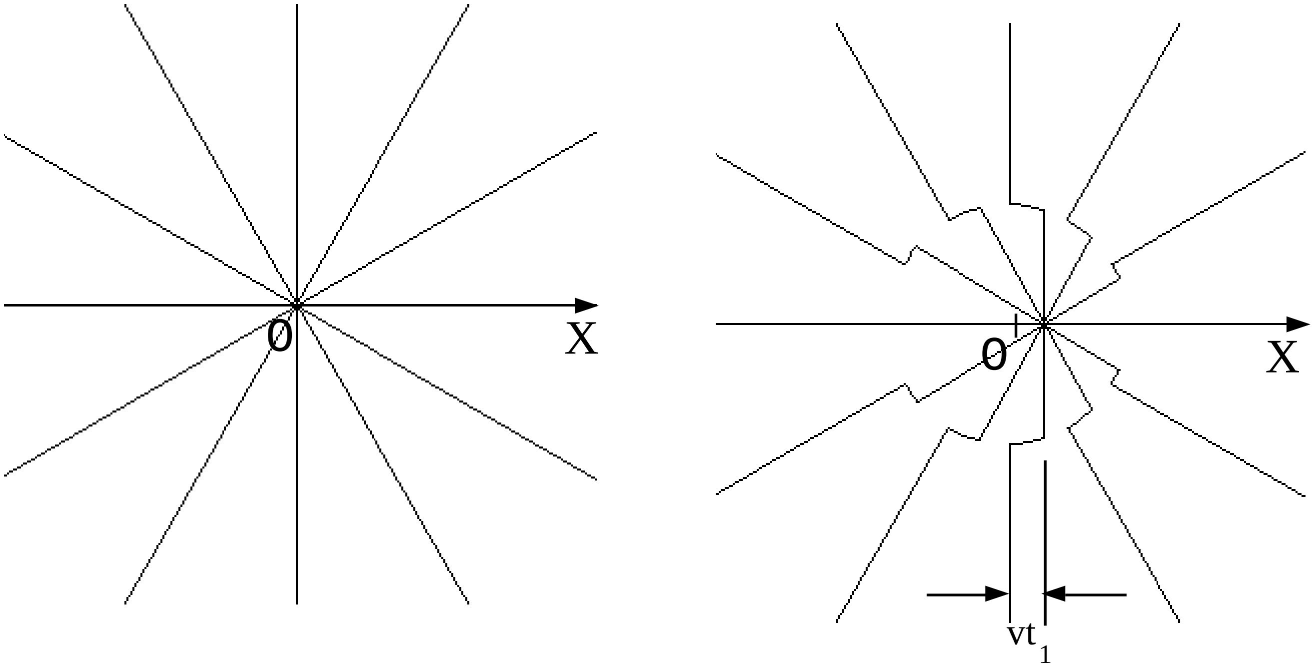
This means that information about any "disturbance" of the position of the charges that produce electric fields, or of the velocity of charges in motion that produce magnetic fields, can not be detected by an observer at a distance R away until a time t = R/c has passed. In EM parlance, this delay is called "retarded time" as that distant observer "finds out" about changes in charge position and/or velocity at a time retarded by the propagation delay. As a simple example, an eruption on the surface of the sun, from which the earth is about 9 minutes away at the speed of light, at a time t_o can not be observed until later at the time t_o + 9 minutes.

Electric Lines of Force Are Continuous--The other fact is that electric charges produce continuous lines of force which in turn define electric fields. A positive test charge placed in an electric field moves along, and in the direction of, the electric fields lines. By convention, electric fields originate on plus- and terminate on minus-signed charges. For charge distributions that are stationary or moving at a constant velocity, the field lines are spatially unchanging and there is no radiation. It's only when the charges are changed in velocity, i.e. accelerated, that radiation occurs, a fundamental property of EM fields that once appreciated can make the mathematically complicated topic conceptually must easier to understand.

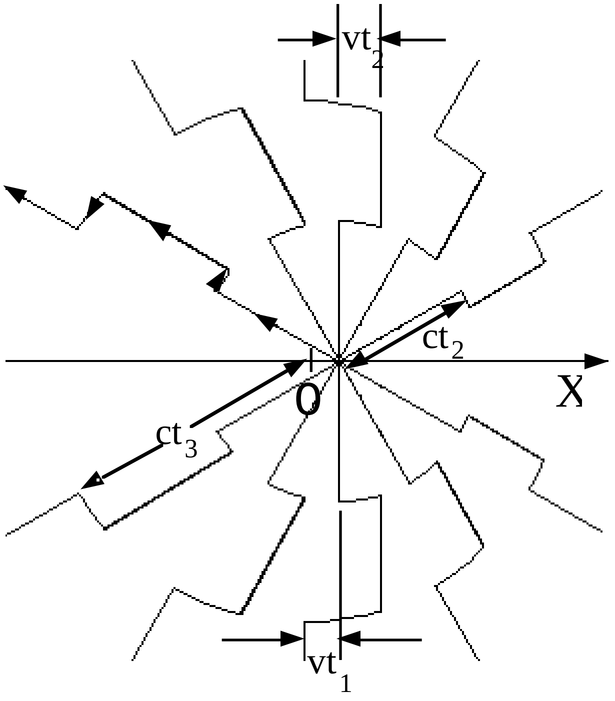
THE "KINK" MODEL OF RADIATION

That radiation occurs only as a result of charge acceleration can be demonstrated by putting together these two simple facts of a finite propagation speed and continuous electric-field lines. Consider a stationary, isolated charge at the origin, O, of a rectangular coordinate system, whose electric field lines therefore lie along radii terminating at O, as shown in Fig. 1a. If the charge is abruptly accelerated to a velocity v = 0.3c along the +x axis, then at time t = t_1 the field lines will be as shown in Fig. 1b. An observer in the x-y plane at point $R = \sqrt{(x^2 + y^2)}$ won't be aware of the charge movement until a time t = R/c has passed when the electric field at the observer's position changes. This change occurs because the E-field lines that terminated initially on the charge at the origin must "shift" over time with the changing charge position, a process that is not instantaneous because c has a finite value. Also, because of the continuity in the E-field lines, the outer part of a given line will continue point to the origin while that part near the charge will point to its changing location. Thus, the old and new field lines are joined by a line segment (or kink) that is parallel to neither, but which lies along the circumference of the sphere (a circle in this two-dimensional plot) defined by R = ct and which moves outward at the speed of light. These kinks constitute the electric components of an electromagnetic radiation field which are accompanied by magnetic-field components as well, which are not shown here. If the charge is abruptly stopped at time t_1, a second spherical surface of E-field kinks is formed as shown in Fig. 1c at a time t_2 later. This new surface propagates outward from the position $x = vt_1$ about which it has expended to the radius $R = ct_2$ while the original radiation pulse has further expanded to a radius $R = c(t_1 + t_2) = ct_3$.

There are some other interesting phenomena to observe in Fig. 1. Note that the E-field lines joining the two kinks continue to move in the +x direction at the speed v to continue pointing at the location where the charge would be had it not been stopped. In addition, observe that the kink portions lengthen as their respective spheres propagate further away from their origins, indicating a decreasing value of their electric fields, which must decay with distance, r, as 1/r. Finally, note that the distance between the acceleration and deceleration kinks in Fig. 1c is shorter in the +x direction than in the -x direction. This demonstrates the well-known Doppler shift exhibited by a moving source of electromagnetic or acoustic energy, i.e., a "squeezing" together of the waveform in the direction of motion and its "stretching" out in the opposite direction..



(a) (b)



(c)

Figure 1. Depiction of the E-field lines for an initially stationary charge (a) that's abruptly accelerated from the origin to a speed v along the positive x-axis (b) and after a time t_1 is abruptly stopped (c). Information about the changed charge positions propagates outward from its old and new positions at the speed of light. This is shown by the circular line segments joining the original field lines and those moving with the charge (b) and the moving field lines and those that terminate at the new stationary position (c). The direction of the radiated field caused by charge acceleration is opposite that due to deceleration as can be seen by the reversed directions of the kinks on the field line with the arrows in this plot.

Propagating Fields Are Vectorial--It's a fact, perhaps unfortunate from a vector-calculus viewpoint, that EM radiation is a vector phenomenon. To see how this is so, now consider an observer located at $x = X$, $y = z = 0$ and retrace the steps just taken. Because the charge is moved along the x-axis, the old and new field lines at the observer's position also lie along the x-axis. This means there is no "joining" non-radial segment needed to connect the old and new lines, and so there is no propagating, or radiating, component of electric field. As a matter of fact, a short, straight (compared with the wavelength) segment of current has a maximum in its radiated electric field perpendicular to its axis and a zero along its axis. By a principle know as reciprocity, its receiving characteristics will have a similar orientation dependence. Furthermore, the radiation components of the electric field lie in the plane of the current segment, and being tangential to the expanding circle are therefore vectors.

A practical consequence of this vector nature of EM radiation is that an antenna made of a straight piece of wire such as typified by a "whip" antenna on a car will receive a maximum signal when oriented parallel to an AM broadcast antenna. Conversely, it will produce a "null" or zero signal when it points at the broadcast antenna. These whip antennas are normally mounted vertically on a vehicle since the stations from which they receive their signals are installed perpendicular to the earth's surface.

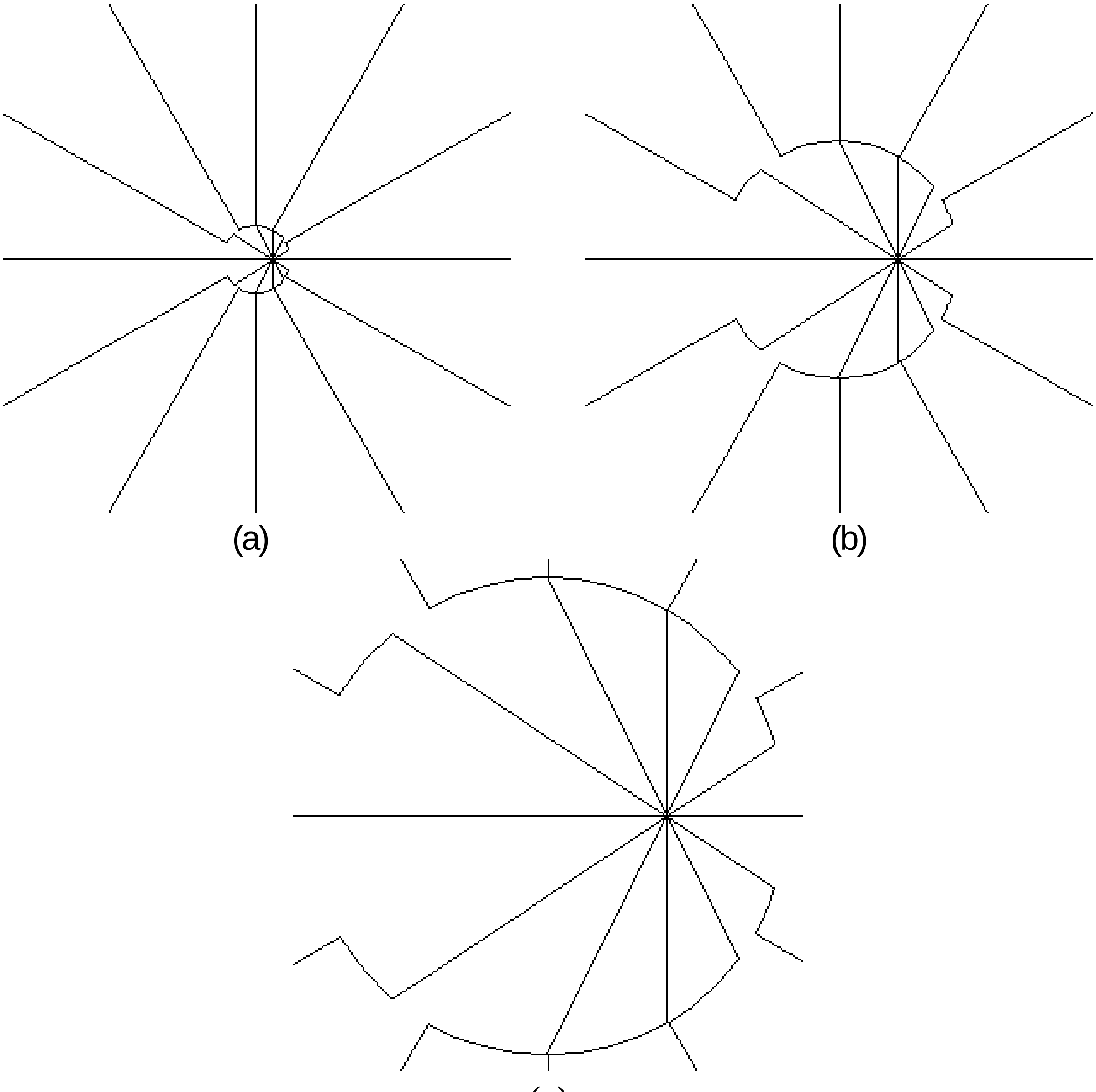
Note that while this conceptual demonstration of the "kink" radiation model is qualitatively correct, it ignores relativity, the effect of which must be taken into account to produce a more accurate picture of the E-field lines. The results shown in Fig. 1 do not explicitly account for the time required to move the charge from $x = 0$ to $x = d$, a time that would be greater than d/c since no physical object can move at light speed. The results that follow do include relativistic effects, although for the cases shown these effects are not very dramatic.

SOME SIMPLE CHARGE MOTIONS THAT PRODUCE RADIATION

The radiation kink model, although not useful for solving boundary-value problems in EM, the solutions of which are needed for determining the scattering properties of radar targets or the current distributions on actual antennas, is never-the-less very helpful for illustrating various kinds of radiation phenomena. A computer program based on the kink model was developed at Stanford University about 1985. This program develops a series of E-field plots for a point charge undergoing certain kinds of motion selected by the user. These plots can then be viewed as a movie to display the evolution of the field as a function of time in a manner similar to a continuous "film loop." Some sample plots made by that program are presented here to show how simple the radiation process actually is, in contrast with the mathematical complexity needed to describe it rigorously. In the results shown, time advances in proceeding from part (a) to (b) to (c) of each figure.

A Charge Given an Abrupt Push--An abrupt push is perhaps the simplest kind of radiation-producing acceleration to visualize, for which an example is shown in Fig. 2. This example differs from that depicted in Fig. 1 where, for simplicity, the charge was assumed to be stationary before and after the acceleration; here the charge continues in motion after receiving an impulse of acceleration.

A single burst of radiation is also produced in this case, but because the final charge speed chosen here is 0.5c, a relativistic effect is seen as a "bunching" of the field lines towards a line perpendicular (the y-axis) to the direction of motion. The faster the charge speed, the more the field lines bunch around the perpendicular. A practical use of impulsive acceleration occurs when an electron beam impinges on a physical object, producing what is known as Bremsstrahlung (breaking) radiation. High-energy electrons smashing into a dense material such as tungsten are used to produce X-rays through this phenomenon.


(a) (b)

(c)

Figure 2. E-field lines about a charge undergoing impulse acceleration as obtained from Cabrera’s Radiation program. The charge starts from rest and reaches a maximum speed of 0.5c to produce the "bubble" of radiation shown expanding from (a) to (c).

A Charge Given a Constant Push--Suppose that instead of being impulsively accelerated to some maximum velocity, a charge is continuously accelerated. While relativity theory shows that the charge's final speed can't exceed that of light, its speed can approach c, increasing in mass as it does so. As seen in Fig. 3 where the final speed is 0.99c, the radiation has the appearance of an EM shock-wave with a zone of nearly overlapping field lines (the density of the lines is a measure of the field strength) which is most concentrated towards the forward direction. This kind of radiation is what would be associated with a linear particle accelerator where charged particles are accelerated to speeds near c over extended distances.

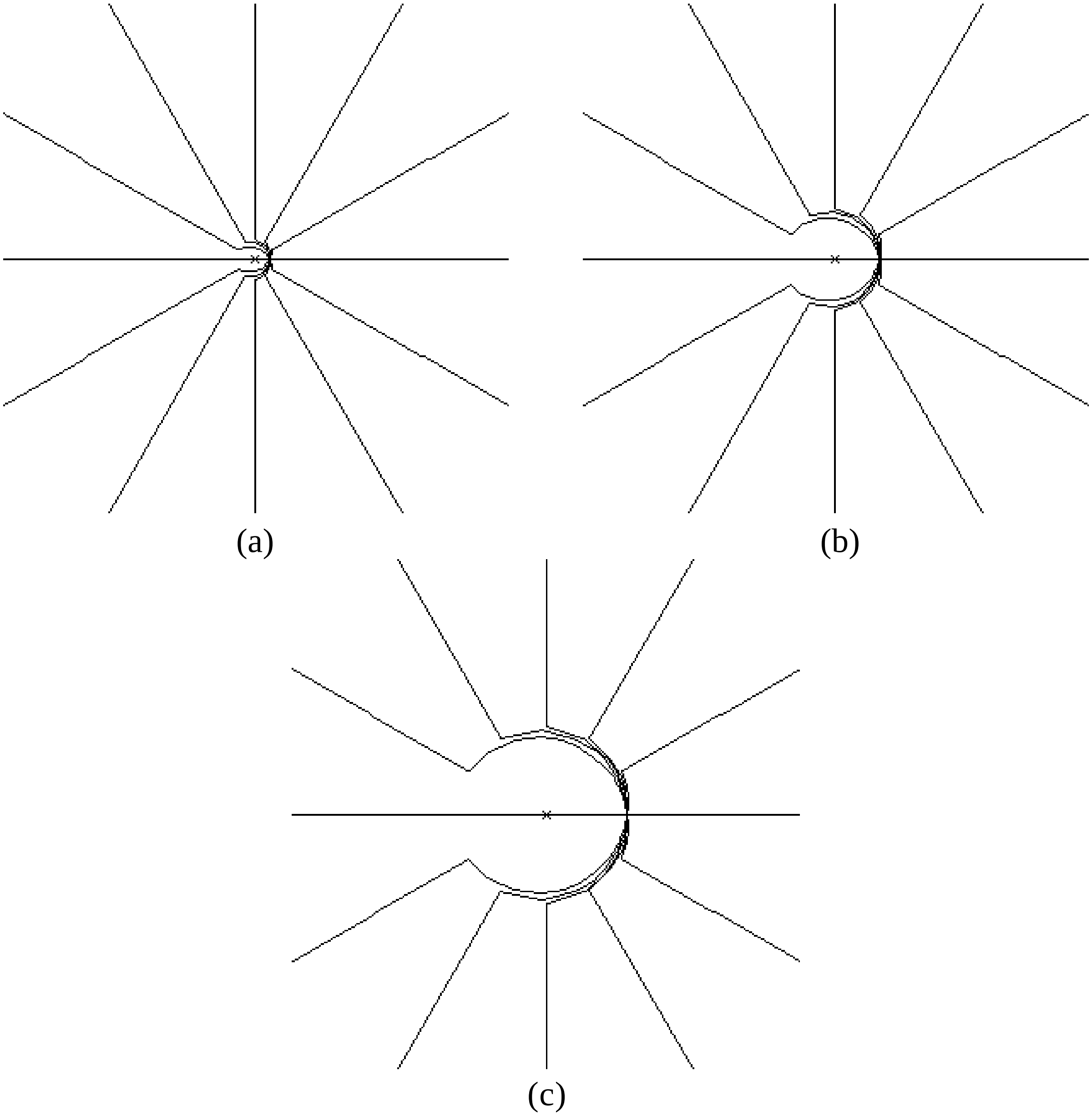
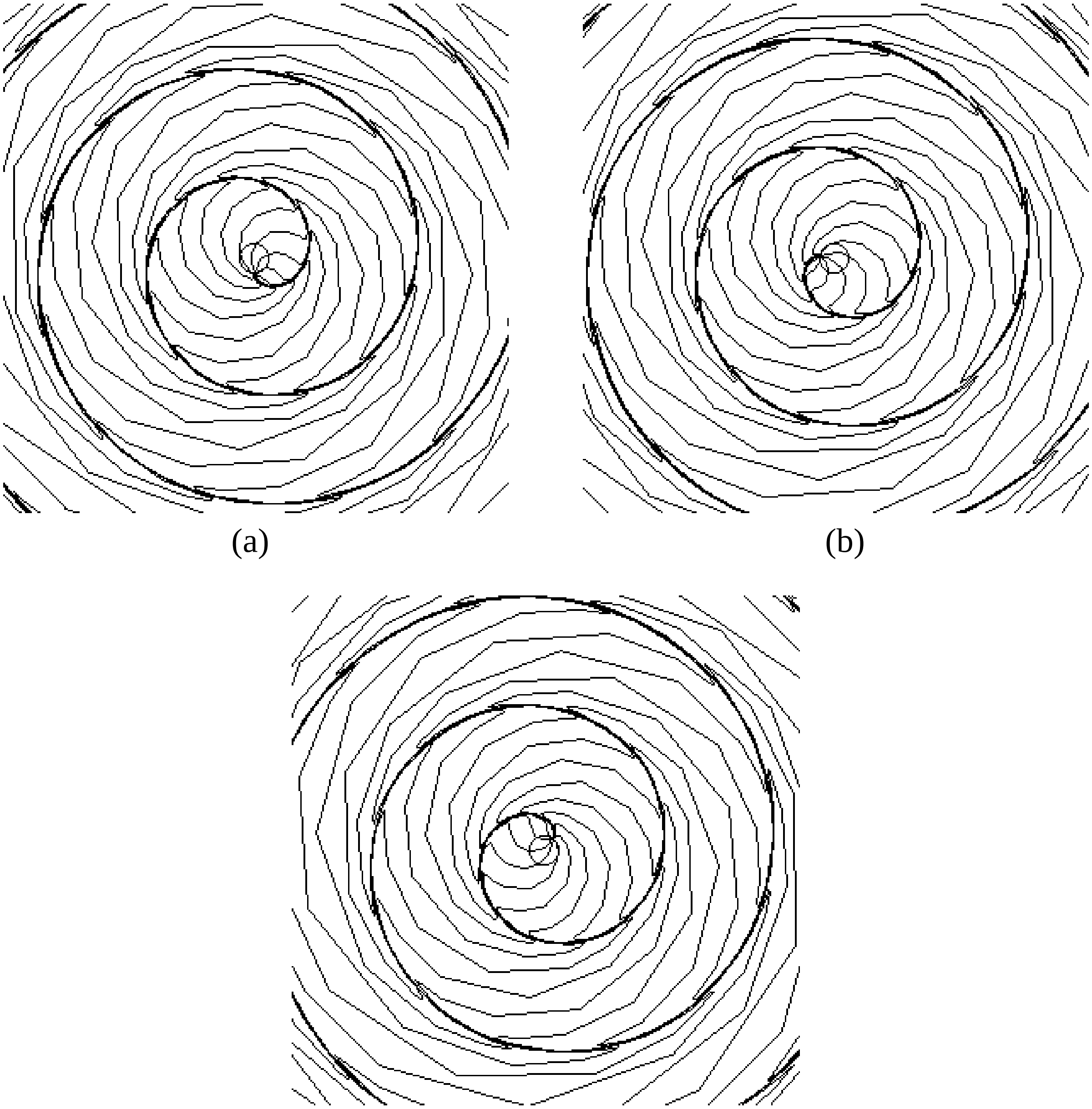

(a) (b) (c)

Figure 3. E-field lines for a charge accelerated to 0.99c by a constant force as obtained from Cabrera's program. Because its speed is so near c, the field lines at the charge are concentrated in a small angular sector about the perpendicular, creating an EM shock wave.

A Charge Moving at Constant Speed Around a Circle--Results are shown in Fig. 4 for a charge moving around a circle at a constant speed of 0.9c. There the strongest radiation field can be seen as a spiral of coalescing field lines synchronized with the motion of the charge around the circle. This is the kind of radiation produced in circular particle accelerators, and is called "Synchrotron" radiation. It's also sometimes called searchlight radiation, as a burst of radiation is seen at a given observation point with every orbit of the charge, much like the flash of light produced by a continuously rotating searchlight beam. The radiation takes place continuously here since the charge is constantly inwardly accelerated as it moves around the circular path, in contrast with the linear acceleration of the previous examples.



(a) (b)

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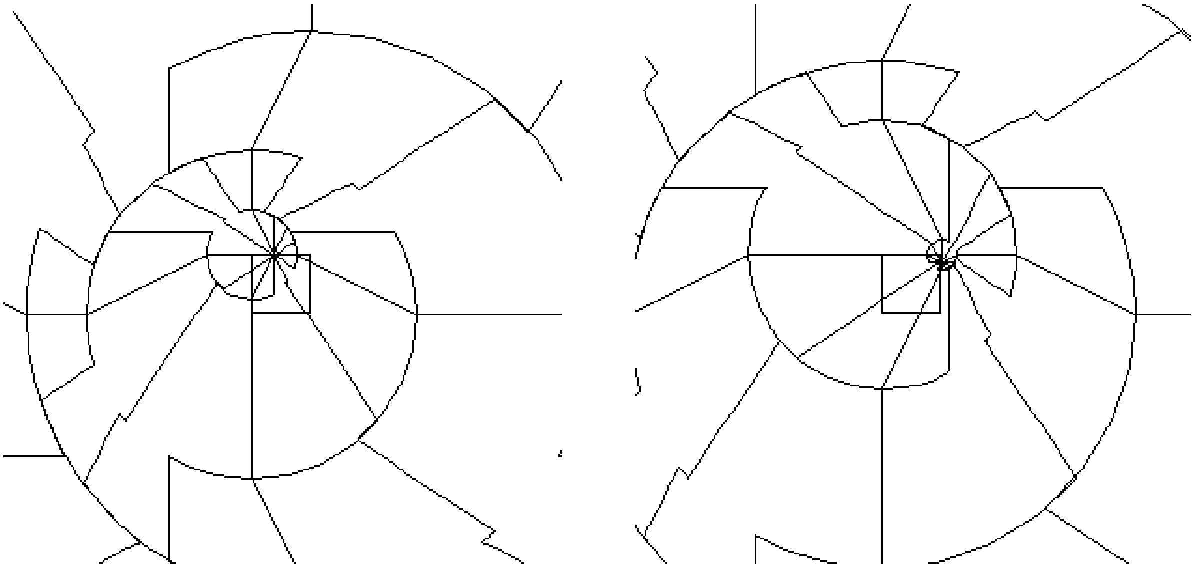
Figure 4. E-field lines for a charge moving in a circular path at a speed 0.9c as obtained from Cabrera's program. High values of electric field are indicated by the dark bands in this figure, producing an effect called Synchotron radiation.

A Charge Moving at Constant Speed Around a Square--Results for a charge moving at a constant speed of 0.5c around a square path are shown in Fig. 5. This kind of charge motion can be especially illuminating since the radiation occurs in a series of pulses as the charge moves around each right-angle corner of the square. During the time the charge moves along the straight side of the square, there is no radiation, but a circular radiation field can be seen centered on each corner as the charge changes direction there. The vectorial nature of this kind of radiation is demonstrated by these radiation pulses not being closed circles, there being nulls in the radiation along a line bisecting each interior angle of the square.



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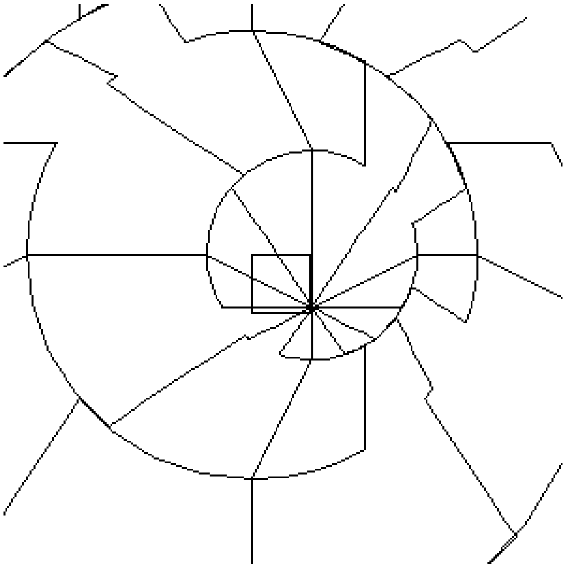
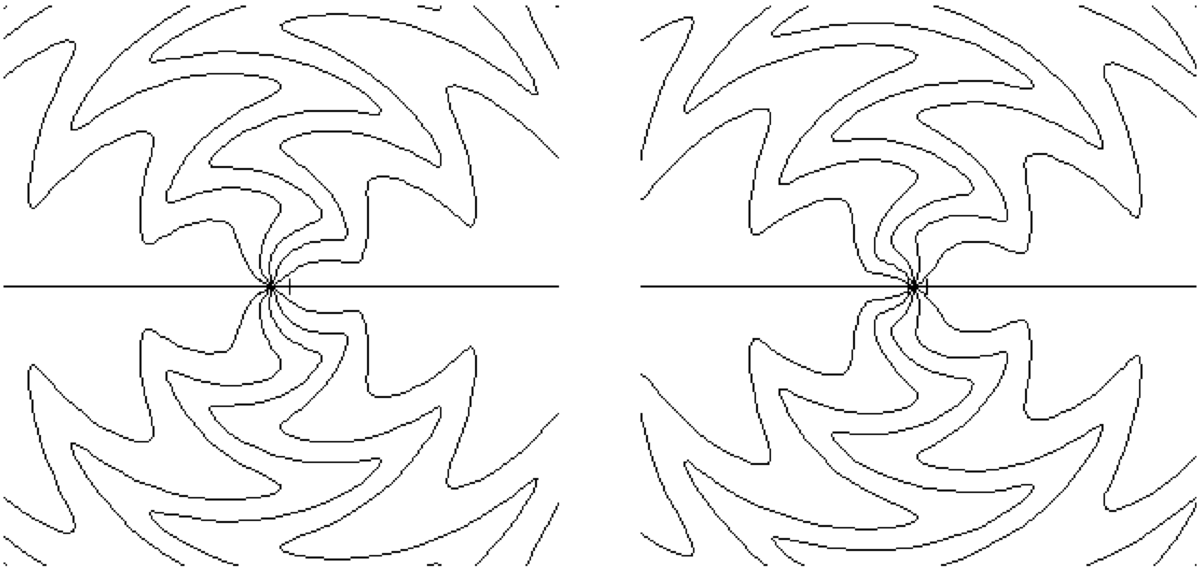


(c)

Figure 5. E-field lines for a charge moving around a square path at a constant speed of 0.5c as obtained from Cabrera's program. The charge produces an expanding circle of radiation as it goes around each of the right-angle corners of the square.

A Charge Undergoing Oscillatory Motion--The kind of motion considered here that is most relevant to electromagnetic communications is oscillatory, where a charge moves back-and-forth along a straight line with speed that varies as Vcos(ωt), with $\omega = 2\pi f$, where f is frequency of the radiation that it produces and V = 0.5c. This is not exactly analogous to how the charge motion varies on an actual wire antenna such as the whip mentioned above, but it suffices to illustrate the production of a time-harmonic radiation field as shown in Fig. 6. Not surprisingly, the radiation field is seen to be oscillatory in space, and therefore in time as well at a fixed observation point, as the wave propagates by.



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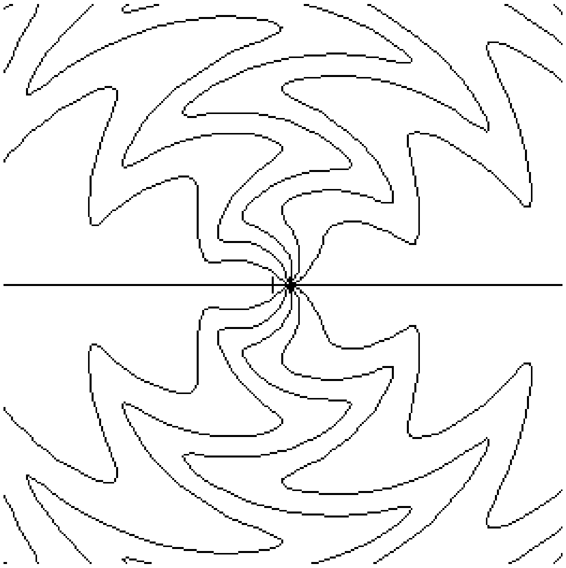

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Figure 6. The E-field lines about a point charge undergoing oscillatory motion along a straight line with a maximum speed of 0.5c as obtained from Cabrera's program. The generation of outward propagating, time-harmonic waves is clearly discernible, in a fashion somewhat analogous to how an actual wire antenna works.[1]

SOME OBSERVATIONS

At this point it's appropriate to point out that a kink model demonstrates that EM radiation is essentially produced by "wriggling" a charge so as to wriggle its E-field lines. As can be seen in Fig. 6, the outward-propagating waves carried by a line of electric field is tantalizingly similar to the waves caused on a rope tied on one end by wriggling the other end. This analogy is not entirely superficial. Both waves are transverse, i.e., the displacement of the field line and the rope are perpendicular or transverse, to the direction of wave travel. Also, a field line in the direction of the wriggling charge motion carries no wave and neither does a rope when its end is pushed or pulled on. Thus, any time a charge or current, which is comprised of moving charges, is wriggled, radiation can be expected.

Electromagnetic radiation also involves power flow. The greater the power provided to an antenna, the larger the field at a distant point where its strength varies as the square root of

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[1] The motion of a point charge and the E-field associated with that motion as illustrated here is quite different from the behavior of charge and current on a real conductor such as a straight wire. For example, on a wire assumed to be perfectly conducting and excited at its center by a time-harmonic voltage, a structure known as a dipole antenna, the effective speed of the E-fields along the wire is c, since EM waves propagate at light speed in free space. Since the EM wave E-field lines terminate perpendicularly at charges on the wire, this means the effective charge speed is also c, which is in contrast to the sinusoidal speed assumed here for the charge. But physical charges in the wire can't move at light speed, and neither does the same physical charge move from one end of the wire to the other. Instead, the charges interact somewhat as a row of falling dominoes to constitute an equivalent current that moves much faster than the charge speeds.

the antenna power. The radiation kink model does not include any consideration of the power needed to make physical charges move in the various arbitrary ways chosen for the above examples. Also observe that power in the EM field is carried by interacting electric and magnetic fields, in an amount and direction given by **ExH** where **H** is the magnetic field and "**x**" signifies a vector cross product. The magnetic field has been neglected in this discussion for simplicity.

Finally, EM radiation and scattering problems normally begin without knowledge of what the charge and current distributions may be on objects of interest, say that vehicular whip antenna previously mentioned. Before the radiation fields can be obtained, it's necessary that the currents and charges flowing on the whip antenna and the vehicle on which it's mounted be found. This in turn requires solution of what is called a "boundary-value" problem, where use is made of the fact that an electric field tangential to a good conductor such as aluminum or copper is zero. When this "boundary condition" is used together with a specification of how the antenna is "excited" by either an incident EM wave when used in a receiving system, or by the voltage from a generator when used in a transmitting system, the currents and charges can be obtained. These in turn permit evaluation of the receiving or transmitting properties of the antenna. This procedure for solving these kinds boundary-value problems can be complicated. The kink model, though unsuited for quantitative solution of such problems, can yield insight into electromagnetic radiation that a more rigorous approach may not provide.

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